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A Cumulative Prospect Theory Approach to Option Pricing

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Abstract

It is a well known empirical fact that actual option prices show persistent and systematic deviations from Black-Scholes option values. While a substantial number of enhancements have been proposed in the literature, these approaches typically leave investors' preferences towards risk unmodified. In this paper we study option prices in an economy where investors are valuing call options according to the cumulative prospect theory of Kahneman and Tversky. We distinguish two prospect option pricing models, based on whether cash flows are either considered to be segregated or aggregated over time. These models are compared with the Black-Scholes model and the stochastic volatility model of Heston. Empirical analysis of European call options on the S&P 500 index shows that prospect option pricing models significantly improve the fitting performance compared with the Black-Scholes model and that especially the aggregated version's performance is at least equivalent to the Heston model.

1. Introduction

Pricing behavior in financial markets has traditionally been explained by assuming rational investors. They correctly update their beliefs when they receive new information and given their beliefs they make choices, which maximize their expected utility. In the past couple of decades, however, the finance literature has amassed a substantial number of observations of apparent anomalies with respect to expected utility theory. Empirically, prices might deviate from those predicted by the expected utility framework, because of the irrational behavior of market participants[1]. During the 1990s, finance literature started to re-explore psychological concepts to explain the behavior of market participants as a separate field of research. Most of the behavioral finance research has been focused on stock markets and attempts to explain among others phenomena like under- and overpricing, hypes and panic, premium between risky and risk-free assets, preference for cash dividends and the tendency to sell winning stocks rather than losing stocks. Overviews of the various aspects of behavioral finance have been given by Barberis and Thaler (2003), Goldberg and von Nitzsch (2001), Shefrin (2002), Shiller (1999) and Thaler (2005).

Within the traditional finance paradigm, the most frequently used model to determine the price of options is based on the theory of Black and Scholes (1973). This theory is based on a riskless portfolio of stocks and a call option on that stock. Empirically, however, prices of traded options systematically deviate from the value calculated by the Black-Scholes theory (for an overview, see Mayhew, 1995). One explanation for this deviation is to question the assumptions of the theory. Another explanation is based on the fact that the Black-Scholes portfolio is not

[1] Interestingly, the basic idea was already launched as early as 1759 in Adam Smith's book about "The Theory of Moral Sentiments" (Nava, Camerer and Loewenstein, 2005).

riskless (Omberg, 1991) and therefore behavioral aspects of (potential) option investors do interfere with the pricing process of an option.

At the empirical level, several studies have shown that option pricing models with conditional heteroskedasticity and negative correlation between volatility and spot returns capture the particular mispricing pattern and significantly improve upon the performance of the Black-Scholes model. The discrete-time GARCH option pricing model has shown to be a flexible, empirically successful model (see among others Heynen et al (1994), Duan (1996), Heston and Nandi (2000) and Bams et al. (2009)). Recently, an increasing number of simulation and numerical methods for this class of option pricing models become available (see Duan and Simonato (1998), Ritchken and Trevor (1999), Heston and Nandi (2000) and Duan et al. (2001)). Using a generalized GARCH option-pricing framework, Lehnert (2003) showed that conditional leptokurtosis and skewness reinforces the effects of conditional heteroskedasticity and asymmetry in the volatility process. His GARCH option pricing model driven by skewed generalized error distributed innovations outperforms the closed-form GARCH option pricing model of Heston and Nandi in-sample as well as out-of-sample. The improvements in pricing errors are particularly pronounced for out-of-the money put and call options, while the model partly underperforms the Gaussian model for near-the-money options. The results are in line with recent results obtained by Christoffersen et al. (2006). While they demonstrate the importance of conditional skewness and jumps for the pricing of out-of-the-money puts, their closed-form Inverse Gaussian GARCH option pricing model significantly underperforms a standard Gaussian model for several other types of options. Therefore, the empirical evidence does not necessarily suggest that e.g. modeling jumps in returns and volatility is the appropriate approach for the purpose of option pricing. In recent years, it becomes apparent that the mispricing for some types of options is only marginally improved, but those models cannot adequately account for the particular pattern observed in option prices.

On a less sophisticated level, partitioning of volatility with respect to moneyness and maturity also diminishes the mispricing of options with the Black-Scholes model significantly (Dumas et al. (1998)). They propose so-called implied volatility functions to capture the observed implied volatility surface of traded options, but do not give any behavioral explanation. As volatility is related to the underlying asset and not to a specific option series, it should be concluded that either the assumption in the Black-Scholes theory that the underlying is a geometric Brownian process is not correct or that investors hold different preferences for different option series. If we consider the first explanation with respect to stocks and indices, then it might be true that volatility is dependent on maturity because future return distributions of short horizon are generally leptokurtic, while higher horizon returns are normally distributed (Fama, 1965). It is therefore hard to imagine that volatility will have an effect on the actual prices within an option series of equal maturity with a longer horizon than one month. Therefore we will focus on the second explanation, the one that assumes that deviations between actual option price and Black-Scholes model predicted value are due to behavioral aspects.

The first 'behavioral' explanation of option pricing is due to Shefrin and Statman (1993). They assumed prices of call options in a one period binomial setting and compared value function[2] modified option prices with prices from the CRR model (Cox, Ross and Rubinstein, 1979). Particularly Shefrin and Statman focused on covered calls: writing call options on stock in possession and concluded that the perceived value and consequently the choice from it, is highly dependent on the parameters of the value function. They find that this is especially the case for

[2] The authors use the Tversky and Kahneman (1992) type of value function.

investors who are sufficiently risk-averse in the domain of gains[3]. Analysis from the viewpoint of the writer is plausible. According to a 1992 Philadelphia Stock Exchange survey, writing of call options as covered calls is the most important objective of investors (Shefrin and Statman, 1993). And non-market maker investors have more written option positions than bought positions (Lakonishok, Lee, Pearson and Poteshman, 2007).

Shefrin and Statman do not consider the weighting function of the prospect theory in their analysis. However, according to Shiller (1999) it is particularly the weighting function that may explain mispricing of options. Breuer and Perst (2004) included a (competence modified) weighting function to determine the prospect value of discount reverse convertibles (DCRs). DCRs can be interpreted as a combination of a risk free asset with a short position in put options. Prospect values based on the Tversky-Kahneman values for its parameters, are compared with the Black-Scholes model in a multi period continuous time setting. Breuer and Perst conclude that investments in risk free assets are preferred in low drift stock markets; in higher drift stock markets the investment in stock is preferred, while in medium drift stock markets DCRs are preferred with low volatility and risk free assets with higher volatility prediction for the price of the stock.

Using the same set of parameter values for all agents suggests that they have the same attitude towards gains and losses. In their experimental study based on a one period binomial CRR estimation problem, Abbink and Rockenbach (2005) find a remarkable difference in the option pricing behavior of professional traders and students. Investors are assumed to follow an option valuation strategy with a separating price and will buy the option whenever the option price is lower than the separating price and will sell the option if the option price is higher than

[3] De Groot and Dijkstra (1996) re-analyzed value function modified covered calls in a dynamic setting using monthly returns on a Dutch equity index and came largely to the same conclusion as Shefrin and Statman.

the separating price. At small probabilities professional traders have a significantly higher separating prices compared to students. At large probabilities both traders and students have equivalent separating prices, which are higher than the separation price at low probability. Their results suggest that it remains questionable whether it is realistic to generalize Tversky-Kahneman results with respect to the values of the parameters to real world trading processes.

Also in ‘markets’ that are not liquid, like investment projects, specific behavioral biases affecting the subjective valuation of (real) options have been reported. Howell and Jägle (1997) show that the average of the assessments of real options by managers tends to deviate erratically from the normative Black-Scholes model. Miller and Shapira (2004) find that buyers and sellers price options below their expected values with buyers’ prices consistently below sellers’ prices.

Another way of looking at the behavioral aspect of option pricing is based on agent theory in which economic agents behave according specified beliefs and therefore have different views about the value of a financial proposition. Guo (1998) considered two agents with different expectations with respect to the values of the parameters of the underlying geometric Brownian price process and was able to perfectly fit prices of a small sample call options on the S&P 500 index. The parameter values of the two agents from his analysis, however, are so far apart, that it is questionable whether these agents are representative for real world traders. Another two agent model due to Benninga and Mayshar (2000) shows that different expectations with respect to relative risk aversion might be another explanation for the observed mispricing of options.

In a recent paper, Poteshman and Serbin (2003) analyze the early exercise of Chicago Board Options Exchange listed call options by different classes of investors over the 1996-1999 period. They find support that there are a large number of early exercises that can be identified as clearly irrational without invoking any model of market equilibrium, and these exercises are not uniformly distributed across the investor classes. Irrational exercise is triggered both by the underlying stock price attaining its highest level over the past year and by the underlying stock having high past returns. Their findings provide evidence that prospect theory is operative in the options market and that it applies differentially across various classes of investors. In a related study, Blackburn and Ukhov (2006) investigate the shape of investor's utility function. Using options on the stocks in the Dow Jones Index, they find support for non-concave utility functions with reference points as proposed by Kahneman and Tversky. The evidence for the value function used by Kahneman and Tversky is much stronger than the support for the standard concave utility function.

In this paper, we study option prices in an economy where the marginal investor values options according to the prospect theory of Tversky and Kahneman. The design of our pricing model includes behavioral aspects like risk attitude, mental accounting and probability perception. Using prospect theory allows us to simplify a problem into mental accounts. Rockenbach (2004) compares three different mental accounts with the CRR model in a one period dichotomy setting and concludes that in a controlled classroom experiment mental accounts are a more realistic explanation of price estimation behavior than the CRR model, indicating the importance of mental accounting in the pricing of options.

The paper is organized as follows. Section 2 introduces three important concepts relevant to the behavioral aspects of option pricing: 'framing', 'mental accounting' and 'prospect theory'. In the next section, the concepts are applied to option pricing to establish models to estimate marginal prospect-based prices. A numerical example is provided to highlight the effect of different degrees of prospect behavior on option prices. Section 4 deals with an empirical analysis of European call options on the S&P 500 index, including prospect-based option pricing, pricing based on the Black-Scholes formula and pricing based on the stochastic volatility model of Heston. Finally we wrap up with a conclusion section.

2. Behavioral aspects of option pricing

2.1. Framing

Framing refers to the way a problem or proposition is presented. It is therefore objective to the subjects that take decisions based on the presented problem. In other words all subjects face the same setting within which the problem or proposition is being described. The presenter of the problem has the choice how to frame the problem; another presenter might frame the problem differently. With respect to option pricing or subjective option valuation one can frame propositions as one period dichotomy prospect, multi period dichotomy prospect, prospects with continuous outcomes or prospects based on the actual options market data. One period dichotomy prospects are generally used in controlled laboratory experiments. They, however, have the drawback of being oversimplified and might be interpreted as a lottery rather than as the outcome of the actual dynamic processes of the stock market. Option traders interpret probabilities differently when exposed to a risky project, where objective probabilities are known or to uncertain prospects, where a subjective assessment of probabilities is required (Fox, Rogers and Tversky, 1996). In this paper we will deal with the pricing of financial options in a real market setting.

2.2. Mental accounting

Mental accounting attempts to describe the process whereby people code, categorize and evaluate economic outcomes. Mental accounting theorists argue that people group their assets into a number of non-fungible mental accounts or mental compartments (Thaler, 1980, 1985, 1999; Tversky and Kahneman, 1981). The decision process consists of two stages: editing and evaluation. In the first stage, people breakdown complex problems into simpler sub problems (Kahneman and Tversky, 1979; Tversky and Kahneman, 1981). In doing so, they apply rules of thumb, or heuristics, that facilitate the interpretation of the various possibilities from which they have to choose. After the various prospects have been edited and categorized, they are evaluated in the second stage of the decision process. The prospect with the highest value is chosen. The rules of thumb when editing and evaluation are necessarily a simplification.

Mental accounting is a subjective process. In other words, it is the interpretation of economic propositions by individuals or groups. The human mind simplifies the real world problem. Therefore mental accounts are generalizations and they tend to be as simple as feasible. Confusingly the editing stage of the mental accounting process is also called the framing stage. In this paper we will refer to 'framing' as the way the problem is presented and to 'mental accounting' as the way the problem is subjectively interpreted.

There may be a range of potential mental accounts for any particularly framed problem. First of all it is dependent on the way the problem is framed. Obviously, if the decision taker is presented a complex problem he will probably use a different way of mental accounting than if he is presented with a simple problem. For instance, if an option valuation proposition is presented in a dynamic continuous world the creation of a mental account is much more difficult than if the problem is presented as a dichotomy one period proposition.

Previous experience is also important in valuing option related problems. A scholar that is well familiar with the Black-Scholes theory values an option by applying the Black-Scholes model, while individuals with no prior Black-Scholes theoretical knowledge value an option by other (heuristic) means, for instance by considering the future density function of the price of the underlying asset or by considering potential future cash flows. Substantial differences in estimation behavior of option values have been noticed between traders and students (Fox, Rogers and Tversky, 1996; Abbink and Rockenbach, 2005). This experience-based estimation seems to be in line with competence modification of the weighting function from prospect theory (Kilka and Weber, 2001).

A breakdown of mental accounts into different type of assets or potential propositions of different risk class has been noted. There is a tendency to view economic propositions as individual investments rather than considering the whole portfolio in the decision process (Fisher and Statman, 1997; Shefrin and Statman, 2000). This phenomenon has been called 'narrow framing' by Thaler and he further proposes that from a potential set of mental accounts with different segregation level, generally the most attractive one is chosen (Thaler, 1985). Thaler calls this decision process 'hedonic framing', which might be a helpful guide explaining how people value propositions (Thaler, 1999). Even when an individual uses a 'narrow framing' mental account, there are several possibilities to segregate. Considering options, one might view naked options versus options in combination with its underlying asset: segregation related to the entities considered. Or one might aggregate or segregate current cash flows versus future cash flows, segregation with respect to time. In this paper we will consider prospect theoretical option values in a entity segregated narrow frame mental account setting for two option pricing models, time segregated and time aggregated.

2.3. Prospect theory

In 1979, Kahneman and Tversky launched their prospect theory in what in retrospect showed to be a seminal paper. On the basis of experiments conducted among colleagues and students, they concluded that the theory of expected utility maximization does not hold in practice. According to Bernstein (1996), one of the cornerstones of prospect theory is the cognitive difficulty to fully understand the subject matter, as nature is so complex that it is hard to draw valid generalizations from what we observe. Tversky and Kahneman showed that when subjects are asked to solve a range of choice problems, they evaluate cash flows by gains and losses in an asymmetric way. People were risk averse in situations of winning, while in situations of losing they were risk-seeking. The experiments also showed that respondents are more sensitive to losses than to gains (loss aversion). Another important part of prospect theory is the finding that people's subjective probabilities are typically different from the objective probabilities. Moreover risk attitude, loss aversion and perceived probability might be dependent on the subject's recent cash flow history. After experiencing a financial gain subjects can increase their willingness to accept risks, while prior losses can decrease the willingness to take risks (Thaler and Johnson, 1990). According to Barberis, Huang and Santos (2001), investors become less sensitive to losses after prior gains due to a shift of the reference point of the value function to a lower value. Conversely, after a prior loss investors become more loss averse. Prospect theory includes the behavioral aspects by two functions. The value function controls for risk attitude (risk-averse or risk-seeking) and loss aversion. The weighting function controls for probability perception. In this paper we will apply the cumulative version of the prospect theory (Tversky and Kahneman, 1992).

Cumulative prospect theory is typically based on discrete uncertain outcomes. A prospect or proposition is a set of events. Each of the events has attached a probability that the event occurs and a monetary value as outcome of that event. The 'prospect value' of the set of events (V) is given by

$$V = \sum \pi v(x) \tag{1}$$

with x being the monetary value of the outcome and v the value function to modify the outcomes. π is the function of decision weights and is dependent on the weighting function modified (cumulative) probabilities associated with the outcomes. As decision weights and value function might be different for non-negative and negative outcomes, the prospect value as given in Equation 1 can be broken down into a value based on nonnegative outcomes and one based on negative outcomes, represented by the positive or negative superscripts

$$V = V^{+} + V^{-} = \sum \pi^{+} v^{+}(x) + \sum \pi^{-} v^{-}(x) \tag{2}$$

Suppose that there are n events with a non-negative outcome and m events with a negative outcome. The events are ordered according to their outcomes. The set $\{x_{i:n},p_{i:n}\}$ represents the i^{th} order of the non-negative outcomes. $x_{i:n}$ is the outcome and $p_{i:n}$ is the associated probability. $x_{1:n}$ is the smallest outcome and $x_{n:n}$ is the largest outcome. Negative outcomes are ordered according to the *absolute* values of the outcomes. The set $\{x_{j:m}, p_{j:m}\}$ represents the j^{th} order of the negative outcomes. $x_{1:m}$ represents the smallest absolute value of the outcomes and $x_{m:m}$ is the largest absolute values of the outcomes. Ordering of the outcomes is necessary as the decision weights are based on cumulative probabilities. More specifically the prospect value presented in Equation 2 might be written as

$$V = \sum_{i=1}^{n} \pi_{i:n}^{+} v^{+}(x_{i:n}) + \sum_{j=1}^{m} \pi_{j:m}^{-} v^{-}(x_{j:m}) \tag{3}$$

with the function of decision weights given by

$$
\pi:\begin{cases}\pi_{n:n}^{+}=w^{+}\left(p_{n:n}\right)\\ \pi_{i:n}^{+}=w^{+}\left(\sum_{k=i}^{n}p_{k:n}\right)-w^{+}\left(\sum_{k=i+1}^{n}p_{k:n}\right), \quad 0<i<n\\ \pi_{j:m}^{-}=w^{-}\left(\sum_{k=j}^{m}p_{k:m}\right)-w^{-}\left(\sum_{k=j+1}^{m}p_{k:m}\right), \quad 0<j<m\\ \pi_{m:m}^{-}=w^{-}\left(p_{m:m}\right)\end{cases} \tag{4}
$$

with w being referred to as the weighting function. The Quiggin function - as being used by Tversky and Kahneman (1992) - is most frequently used

$$
w:\begin{cases}w^{+}(p)=\dfrac{p^{\gamma^{+}}}{\left(p^{\gamma^{+}}+(1-p)^{\gamma^{+}}\right)^{1/\gamma^{+}}}\\ w^{-}(p)=\dfrac{p^{\gamma^{-}}}{\left(p^{\gamma^{-}}+(1-p)^{\gamma^{-}}\right)^{1/\gamma^{-}}}\end{cases} \tag{5}
$$

with γ being a constant that controls for the over- and underweighting of small and large probabilities. p is a cumulative probability.

A typical value function with zero reference point is

$$
v:\begin{cases}v^{+}\left(x_{i:n}\right)=x_{i:n}^{a}\\ v^{-}\left(x_{j:m}\right)=-\lambda\left(-x_{j:m}\right)^{b}\end{cases} \tag{6}
$$

in which a, b and λ are positive parameters reflecting respectively risk attitude (risk averse or risk taking) and loss aversion. Tversky and Kahneman (1992) use this value function.

The discrete cumulative prospect theory may be valuable for lotteries or designed class room experiments, but financial propositions, like options are based on continuous outcomes. To consider cumulative prospect theory for continuous outcomes we divide the probability density function of the outcomes, $f(x)$, in slices of equal width Δx. Suppose that there are n nonnegative and m negative slices and the outcome of a slice is equal to $x_{i:n}$ and $x_{j:m}$ for respectively nonnegative and negative outcomes. The attached probability of an outcome is approximated by the probability density function of the outcome multiplied by the width of the slice, $f(x_{i:n})\Delta x$ and $f(x_{j:m})\Delta x$ respectively. From the Equations 3 and 4, we get the prospect value of the i^{th} slice as

$$V_i^+ = \left[w^+(p_i) - w^+(p_{i+1})\right]\ v^+(x_{i:n}) \tag{7}$$

with the cumulative probability (p_i) given by

$$p_i = \sum_{k=i}^{n} f(x_{k:n})\Delta x \tag{8}$$

For continuous weighting functions, Equation 7 can be approximated by

$$V_i^+ = \frac{dw^+(p_i)}{dp_i}(p_i - p_{i+1})v^+(x_{i:n}) \tag{9}$$

Simplifying the derivative of the weighting function by ψ and substitution of Equation 8 into Equation 9 gives

$$V_i^+ = \Psi^+(p_i) f(x_{i:n}) v^+(x_{i:n})\Delta x \tag{10}$$

The prospect value of the nonnegative outcomes then becomes

$$V^+ = \sum_{i=1}^{n} \Psi^+(p_i) f(x_{i:n}) v^+(x_{i:n})\Delta x \tag{11}$$

For continuous outcomes we have $n \rightarrow \infty$. Substitution of $x_{i:n}$ by x in Equation 8 results the cumulative probability of the i^{th} slice as

$$p_i = \int_x^{\infty} f(x)dx = 1 - F(x) \tag{12}$$

with $F(x)$ being the cumulative distribution function of the outcomes.

The continuous cumulative prospect value of the nonnegative outcomes then becomes

$$V^+ = \int_0^{\infty} \Psi^+[1 - F(x)] f(x) v^+(x)dx \tag{13}$$

For negative outcomes the prospect value is determined similarly. Consequently the prospect value based on the continuous outcomes is

$$V=\int_{-\infty}^{0}\Psi^{-}\left[F(x)\right]f(x)v^{-}(x)dx+\int_{0}^{\infty}\Psi^{+}\left[1-F(x)\right]f(x)v^{+}(x)dx,\quad -\infty\leq x\leq\infty \tag{14}$$

3. The Pricing Framework

The price of an option is the outcome of its perceived value by investors. In equilibrium, the marginal investor determines the price. We assume that the marginal investor prices the option according to the valuation of the prospect theory in an entity segregated narrow framing mental account. In other words we consider a naked option. With respect to time we distinguish between segregated and aggregated (potential) cash flows. For written European call options, we receive a cash amount now and have to pay the maximum of S_T-K and zero at expiration. In the segregated case the cash flow at t=0 and the pay-off at t=T are separately valued by the investor. In the aggregated case the investor mentally combines all cash flows into one prospect proposition. [4]

Assume that the marginal investor is writing L European style call options on a non-dividend paying stock or index. Then (s)he receives L times the option premium, denoted by c, and has to pay $L[S_T-K]_+$ at expiration. Assuming that we can invest the cash amount Lc at the risk-free rate of return (r_f), its future value is equal to $Lc\exp(r_fT)$. Then the segregated prospect value of the option proposition – based on Equation 14 - is

$$V_{Segg}=v^{+}\left(Lc\exp\left(r_fT\right)\right)+\int_{K}^{\infty}\Psi^{-}\left[1-F_A\left(S_T\right)\right]f_A\left(S_T\right)v^{-}\left(L\left(K-S_T\right)\right)dS_T \tag{15}$$

[4] In the remainder of the paper ‘time segregated’ and ‘time aggregated’ are abbreviated by ‘segregated’ and ‘aggregated’.

and taking into account the piece-wise linearity of the pay-off function, the aggregated prospect value of the option proposition becomes

$$V_{Agg} = w^{+}[F_A(K)]v^{+}\left(Lc\exp(r_f T)\right) + \int_{K}^{K+c\exp(r_f T)} \Psi^{+}[F_A(S_T)]f_A(S_T)v^{+}\left(L\left(c\exp(r_f T)+K-S_T\right)\right)dS_T$$
$$+ \int_{K+c\exp(r_f T)}^{\infty} \Psi^{-}[1-F_A(S_T)]f_A(S_T)v^{-}\left(L(K-S_T)\right)dS_T \quad (16)$$

In both cases, segregated and aggregated, we can determine the option price at a prospect value of zero by numerical evaluation. For the segregated case we can find a closed form solution by assuming the value function as being presented in Equation 5

$$c = \exp(-r_f T)L^{(b-a)/a}\left(\lambda\int_{K}^{\infty}\Psi^{-}[1-F_A(S_T)]f_A(S_T)(S_T-K)^b dS_T\right)^{1/a} \quad (17)$$

For the aggregated case there is no closed form formula and the option price should be iteratively determined.

In the remainder of this paper we assume that the value and weighting function are given by the Equations 5 and 6, leading to the derivative of the weighting function

$$\Psi = \frac{dw}{dp} = \gamma p^{\gamma-1}\left[p^{\gamma}+(1-p)^{\gamma}\right]^{-1/\gamma} - p^{\gamma}\left[p^{\gamma-1}-(1-p)^{\gamma-1}\right]\left[p^{\gamma}+(1-p)^{\gamma}\right]^{-(\gamma+1)/\gamma} \quad (18)$$

Further we assume that the price process of the underlying asset is geometric Brownian, resulting in a future probability density function of

$$f(S_T) = \frac{1}{S_T\sigma\sqrt{2\pi T}}\exp\left(\frac{-\left[\ln(S_T/S_0)-(\alpha-\sigma^2/2)T\right]^2}{2\sigma^2 T}\right) \quad (19)$$

with α being the instantaneous drift, σ being the volatility, S_0 being the current price of the underlying asset, S_T being the future price of the underlying asset and T being the time of the payoff. The cumulative distribution function of the future price of the underlying asset is

$$F(S_T)=\Phi\left(\frac{\ln(S_T/S_0)-(\alpha-\sigma^2/2)T}{\sigma\sqrt{T}}\right) \tag{20}$$

with Φ(.) being the cumulative standard normal distribution.

4. Numerical example

In order to demonstrate the effect of non-concave utility functions, loss aversion and probability perception on the price of an option, we show option values under different prospect parameters in relation to changes in the strike price. We define 'prospect sentiment' as the deviation of investor behavior compared to the option value according to the Black-Scholes theory. Suppose a geometric Brownian price process of the underlying asset with parameter values S_0=100, α=0.1 and σ=0.2. The risk-free rate of return is equal to 0.1 and the time till expiration of all options is equal to one year. Strike prices, *K*, are varied from 70 to 120. In the numerical example, we consider three levels of sentiment:

- Zero prospect sentiment (linear value function, no loss aversion and no over-/underestimation of small/large probabilities) referring to option values equal to the Black-Scholes formula with prospect parameters *a*=1, *b*=1, λ=1, γ^+=1, γ^-=1 and α=r_f
- Prospect sentiment, based on the observations from prospect theory that investors are risk averse for gains (concave value function), risk taking and loss averse for losses (convex value function), overestimating small probabilities and underestimating large probabilities. The prospect parameter values are based on experimental evidence by Tversky and Kahneman (1992): *a*=0.88, *b*=0.88, λ=2.25, γ^+=0.61 and γ^-=0.69

- Moderate prospect sentiment including the value and weighting functions the prospect theory, but with parameter values reflecting only 10% of the Tversky-Kahneman prospect sentiment: a=0.988, b=0.988, λ=1.125, γ^+=0.961 and γ^-=0.969.

Prospect option prices with respect to the different strike prices under different sentiment are given in Table 1. Tversky-Kahneman parameter values based prospect sentiment gives 1.3 to 3.4 times higher option values than calculated with the Black-Scholes formula. Moderate prospect sentiment gives slightly increased values in the order of 10% compared to the Black-Scholes reference. Under both sentiment scenario's we find segregated option values to be higher than aggregated option values for all strike prices.

[Table 1]

The higher prices under prospect sentiment increase the implied volatility as being calculated by the Black-Scholes option valuation formula; the implied volatility curves are shown in Figure 1. This effect is consistent with observations made by Fleming, Ostdiek and Whaley (1995) and Christensen and Prabhala (1998) that implied volatilities of short-term at-the-money equity index options are on average higher than the realized volatilities over the option's life.

[Figure 1]

The increase is most pronounced at low strike prices resulting in a pronounced volatility skew (or smirk). The pattern is well known for options on stocks and stock indices, particularly after the market crash of '87, where out-of-the-money put options (and therefore in-the-money calls) are typically overpriced relative to at-the-money options (see for example Rubinstein, 1994). Additionally, the more pronounced the prospect sentiment, the steeper the implied volatility curve. The implied volatility plot under moderate prospect sentiment is equivalent to the pattern observed with actual option prices. The Tversky-Kahneman based prospect sentiment results in a shape that is too extreme.

Additionally we consider the sensitivity of the prospect models with respect to the parameters. More risk averse behavior in the domain of gains (in this case smaller values for a) leads to higher call option prices, while more risk taking in the domain of losses (in that case smaller values for b) lead to lower call option prices. The more pronounced the risk attitude, the higher the sensitivity towards the parameters a and b. The effect is decreasing with increasing strike prices, so in-the-money options are more sensitive towards the risk parameters than out-of-the-money call options. A similar effect can be observed for the loss aversion parameter. The more loss avers the option writer (larger values for λ), the higher the call option price. An upward movement in the stock price would be much more painful than a downward movement, and with increasing loss aversion the impact on the option price becomes stronger. The effect decreases as well with increasing strike prices. Segregated option values are more sensitive towards the parameters of the value function than aggregated values.

The sensitivity with respect to γ is less pronounced than the parameters of the value function. The effect is more complicated due to the particular shape of the weighting function. For the segregated prospect value under Tversky-Kahneman sentiment, we find that the more the marginal option writer overestimates low probabilities (the smaller γ), the lower the option price for in-the-money calls, but the higher the option price for (far) out-of-the-money calls, given zero or moderate prospect sentiment. The call option writer is losing money if the out-of-the-money call is getting in-the-money and will be exercised. The same effect is noted for the aggregated prospect value under moderate sentiment. Interestingly, for the other model/sentiment combinations we find that overestimating small probabilities results higher option values. So, a priori it is not possible to predict the sensitivity towards the parameter of the weighting function as it is dependent on the applied prospect model, segregated or aggregated, and on the sentiment.

5. Empirical analysis

The prospect option pricing models are analyzed with premiums (option prices) of end-of-the-day S&P 500 European options (SPX) supplied by the CBOE. We opt to use data of 2002 and 2006, respectively being the year with the highest (26%) and being the year of the lowest realized volatility of the S&P 500 (10%) in the ten year period from 1996 to 2006. Low volume option series with a volume less than 10 contracts per day have been ignored in the analysis. The prospect models are not sensitive to the number of options, we therefore opt to do the analysis for one option contract (L=100).

As reference we use the Black-Scholes model with constant implied volatility for all option series of one particular trading day. As a further reference, we use a continuous time limit, single lag version of the Heston and Nandi (2000) model, the stochastic volatility model of Heston (1993). Stochastic volatility models show good performance with respect to S&P 500 options (Bakshi, Cao and Chen, 1997) and are considered to be a state of the art benchmark for financial time series. The Heston model is explained in Appendix A. The model is analyzed from the view point of the marginal investor, including his/her preferences for the parameter values of the model.

We perform an in-sample and an out-of-sample analysis. In the in-sample analysis, the parameters of the models are estimated at minimum weighted root mean squared error (wRMSE) by the Newton steepest descent method. We follow Bams et al. (2009) and use the absolute pricing error criterion as our general purpose loss function. The wRMSE is volume weighted and determined by

$$wRMSE = \sqrt{\sum_{i=1}^{n} v_i \left(c_i - c_{i,\text{model}}\right)^2 \Big/ n \sum_{i=1}^{n} v_i} \qquad (21)$$

with c_i being the actual option premium, $c_{i,\text{model}}$ being the premium predicted by the model, v_i being the daily trading volume and n being the number of option series of a particular trading day. All prices are given in US dollars.

The average S&P 500 index is 994 in 2002 and 1310 in 2006. US weekly rates provided by Datastream have been used as risk-free rates of return. The average risk-free rate of return is 1.8% in 2002 and 5.0% in 2006. The call price data is broken down into time to maturity intervals and moneyness intervals, in line with the analysis by Christoffersen, Heston and Jacobs (2006). Moneyness (M) is being defined as

$$M = \frac{K}{S_0 \exp(r_f T)} \qquad (22)$$

Table 2 gives an overview of the number of observations and Table 3 gives an overview of the average call prices.

[Table 2]

[Table 3]

The maximum number of observations lies around the at-the-money options. Out-of-the-money option series are more frequent for the year 2002 compared to the year 2006 (on a relative basis). The analysis includes a wider range of moneyness intervals compared to the Christoffersen-Heston-Jacobs analysis. As a consequence also the range in option prices is wider. Given that we always control for volume, we ensure that contracts are liquid and prices are informative.

5.1 In-sample analysis

Parameter values and weighted root mean squared error of the various models from daily analysis are given in Table 4. The fit of the prospect and Heston models with actual market prices is good and better than the Black-Scholes model. All models show a better fit in the low volatility year (2006) compared to the high volatility year (2002). Penalizing the models for the number of implied parameters by Akaike's information criterion leads to the same conclusion about the fitting performance of the models.

[Table 4]

The marginal aggregated prospect investor is risk averse in the domain of gains and risk taking in the domain of losses, in line with the observations by Tversky and Kahneman (1992). For the year 2002 (s)he is loss averse and overestimates small probabilities, also in line with Tversky and Kahneman. For the year 2006, however, we find that the investor is not loss averse, overestimates small probabilities in the domain of gains, but underestimates small probabilities in the domain of losses, which in not in line with the observations by Tversky and Kahneman.

Interestingly the marginal segregated prospect investor is risk taking in the domain of gains, risk averse in the domain of losses, loss averse and has a weighting function that favors low probabilities that the option will become in the money at expiration. This behavior is different from the observations by Tversky and Kahneman (1992), as their experiments showed that decision takers are risk averse in the domain of gains, risk taking in the domain of losses, loss averse and overestimate small probabilities.

There might be two reasons for the difference in the attitude of investors and the observations by Tversky and Kahneman. First of all, the experiments of Tversky and Kahneman are done with students and scholars, while market prices are governed by actions of traders and investors. And it has been observed that the financial option decisions of traders differ from decisions taken by students (Fox, Rogers and Tversky, 1996; Abbink and Rockenbach, 2005). The second reason might be that the Tversky-Kahneman data is based on the median of the estimates by the decision takers, while in the actual option trading the tails of the distribution of the subjective option values of potential investors are more important for the resulting actual option price than the average or median of this distribution.

For the parameters of the Heston model we note that the difference between v_0 and θ is unrealistically large, so it is questionable whether the parameters of the model describe the estimation attitude of the investor properly.

Additionally we have performed a Wilcoxon signed-rank test on daily wRMSE data. The test is distribution-free and does capture the correlation of the performance of the models. The result is given in Table 5 and shows that the aggregated prospect model performs better than all other models at a significance level of 1%. The segregated prospect model is better than the Heston model for the year 2006, but performs equally well in the year 2002. The Black-Scholes model is significantly worse than all other models.

[Table 5]

A representation of the performance of the models with respect to moneyness can be given by the implied volatility surface, for realistic implied volatilities. Using actual option prices and determining the implied volatility using the Black-Scholes formula we get – on the average – the well known smile or smirk pattern. Additionally we can transform the prices predicted by a model into Black-Scholes implied volatilities as shown in Figure 2. In this representation the Black-Scholes model should give – by definition - a flat smile pattern. However, due to the daily differences in option series this is not exactly the case. The aggregated prospect model follows the smile pattern very well. For in-the-money call options, the Heston model gives lower implied volatilities compared to the smile based on the actual option prices, while the segregated option model has higher (for 2002) and lower (for 2006) implied volatilities, compared to actual option prices.

[Figure 2]

The smile is more pronounced if the time till expiration of the option is shorter and flattens with longer time till expiration (see the Figures 3 and 4 for respectively the years 2002 and 2006). In this term structure of the implied volatility, the aggregated and segregated prospect models follow the actual price pattern quite well. The lower the time to expiration, the more the Heston model deviates from the actual prices smile, especially for in-the-money options.

[Figure 3]

[Figure 4]

A more logical representation of the difference of actual option prices and model predicted prices is mispricing, defined by the actual option price minus the model predicted one. Mispricing of the models is shown in Figure 5 for all the data and in the Figures 6 and 7 for the breakdown into the various times till expiration groups.

[Figure 5]

[Figure 6]

[Figure 7]

Mispricing is most severe for the Black-Scholes model. A systematic pattern with respect to moneyness is noted for this model. From far-in-the-money options mispricing increases up till a moneyness of about 0.95. Around-the-money mispricing decreases. In some cases mispricing increases again for out-of-the-money options. The other models do not show any systematic pattern and generally show low mispricing, except for some deep-in-the-money option series.

5.2 Out-of-sample analysis

The out-of-sample analysis is a measure for the forecasting performance of a model. We opt for a one (trading) day out-of-sample analysis in which the parameter values of a model are used to determine the option price of the next trading day. Table 6 shows the in-sample and out-of-sample wRMSE for the years 2002 and 2006.

[Table 6]

In line with the in-sample wRMSE, the out-of-sample wRMSE is lower for the low volatility year (2006) in comparison to the high volatility year (2002). The out-of-sample wRMSE of the Heston and the aggregated prospect model are the lowest and are of the same order of magnitude. The segregated option model gives slightly higher values than the Heston and the aggregated prospect model. The out-of-sample wRMSE of the Black-Scholes model is the highest. A similar conclusion might be drawn from the Wilcoxon signed-rank test given in Table 7.

[Table 7]

Out-of-sample implied volatility surfaces are almost equal to the in-sample ones and are visually not different, leading to equivalent conclusions.

5. Conclusion

In this paper, we study option prices in an economy where investors' decisions are conform to the behavioral aspects of the prospect theory of Tversky and Kahneman. The theoretical marginal prospect investor writes European call options, is risk averse in the domain of gains, risk taking and loss averse in the domain of losses, and overestimates small and underestimate large probabilities of the option expiring in-the-money. Moderate prospect behavior of investors can explain the shape and size of the implied volatility skew, while prospect behavior based on the parameter values by Tversky and Kahneman is too severe compared to a typical implied volatility pattern.

Empirical analysis shows that incorporation of prospect theory in an option pricing model significantly improves the fitting performance of the model with European call options on the S&P 500 index in in-sample as well as in out-of-sample analysis. Further, the analysis shows that the marginal investor's behavior is different from the prospect theoretical observations and is depending on whether cash flows are considered to be segregated or aggregated.

The aggregated prospect model shows the best fitting performance with actual option prices. In an in-sample analysis this model is better than the stochastic volatility model of Heston, while the Heston model performs better than the segregated prospect model, which in turn gives a better fit than the Black-Scholes model. The out-of-sample performance of the models is in line with the in-sample analysis, with exception of the Heston model that performs equivalently to the aggregated prospect model.

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Appendix A: Heston model for option pricing

The Heston (1993) model for option prices is based on a stochastic volatility price process in which S_t, the price of the underlying asset, is determined by

$$dS_t = \alpha S_t dt + \sqrt{v_t} S_t dz_t^S \tag{A1}$$

where v_t, the instantaneous variance, is a CIR process:

$$dv_t = \kappa(\theta - v_t)dt + \omega\sqrt{v_t} dz_t^v \tag{A2}$$

dz_t^S and dz_t^v are Wiener processes with correlation ρ. α is the instantaneous rate of return of the underlying asset, θ is the long run average price volatility, κ is the rate at which v_t reverts to θ and ω is the 'volatility of the volatility'. As the name suggests, ω determines the variance of v_t. Further Heston introduces a behavior component, the price of volatility risk (λ). This parameter is indicative for the relative risk aversion of an investor.

According to Heston, the price of a European call option is

$$c = SP_1 - K\exp(-r_f\tau)P_2 \tag{A3}$$

with S being the current price of the underlying asset, K being the strike price, r_f being the risk-free rate of return and τ being the time till expiration. P_1 and P_2 are 'probabilities' determined by

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{f_j \exp(-i\phi\ln K)}{i\phi}\right]d\phi \tag{A4}$$

with f_j given by

$$f_j = \exp(C_j + D_j v_0 + i\phi\ln S) \tag{A5}$$

and C_j and D_j being defined as

$$C_j = r_f i\phi\tau + \frac{\kappa\theta}{\omega^2}\left\{\left(b_j - \rho\omega i\phi + d_j\right)\tau - 2\ln\left[\frac{1 - g_j \exp\left(d_j\tau\right)}{1 - g_j}\right]\right\}$$

$$D_j = \frac{b_j - \rho\omega i\phi + d_j}{\omega^2}\left[\frac{1 - \exp\left(d_j\tau\right)}{1 - g_j \exp\left(d_j\tau\right)}\right] \tag{A6}$$

with

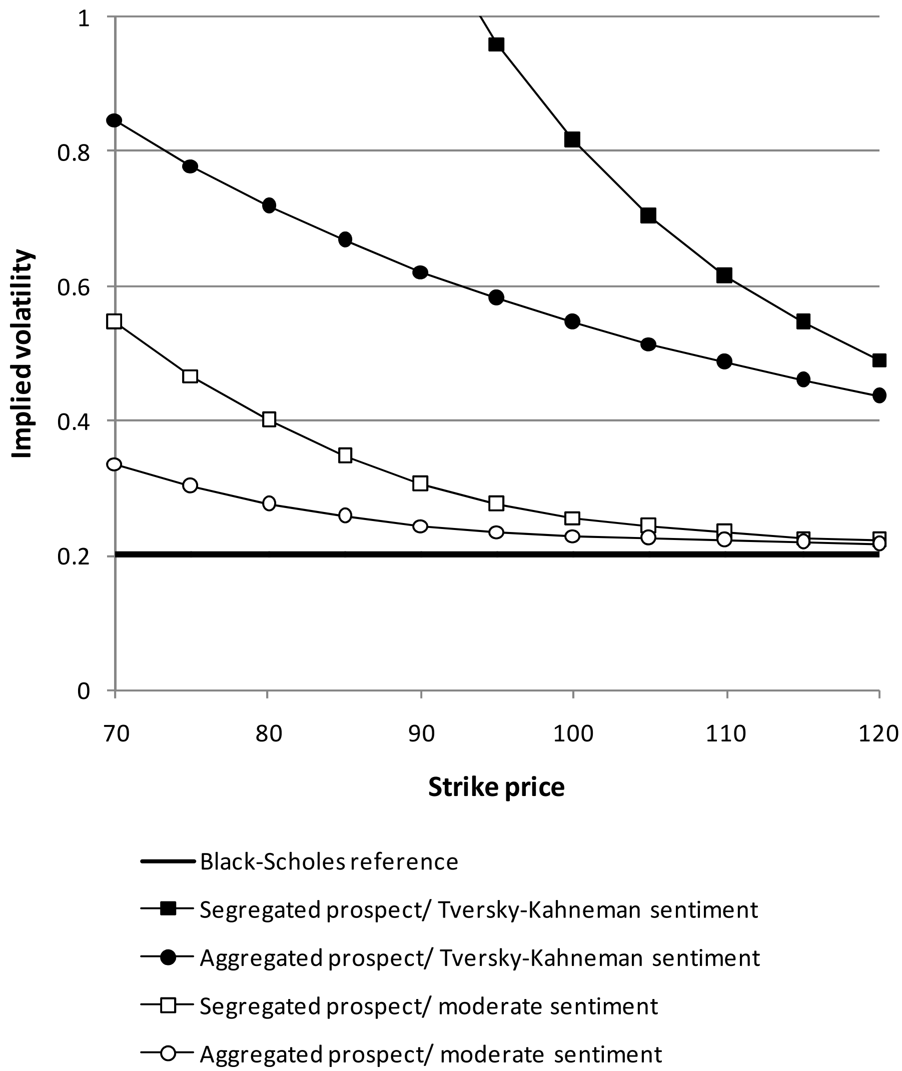
$$g_j = \frac{b_j - \rho\omega i\phi + d_j}{b_j - \rho\omega i\phi - d_j}$$

$$d_j = \sqrt{\left(\rho\omega i\phi - b_j\right)^2 - \omega^2\left(u_j i\phi - \phi^2\right)} \tag{A7}$$

$$b_j = \kappa + \lambda + \left(j - 2\right)\rho\omega$$

$$u_j = 3 - 2j$$

Figure 1: Implied volatility *



* For the parameter values: see Table 1.

Figure 2: Average implied volatility from S&P 500 call options (all data)

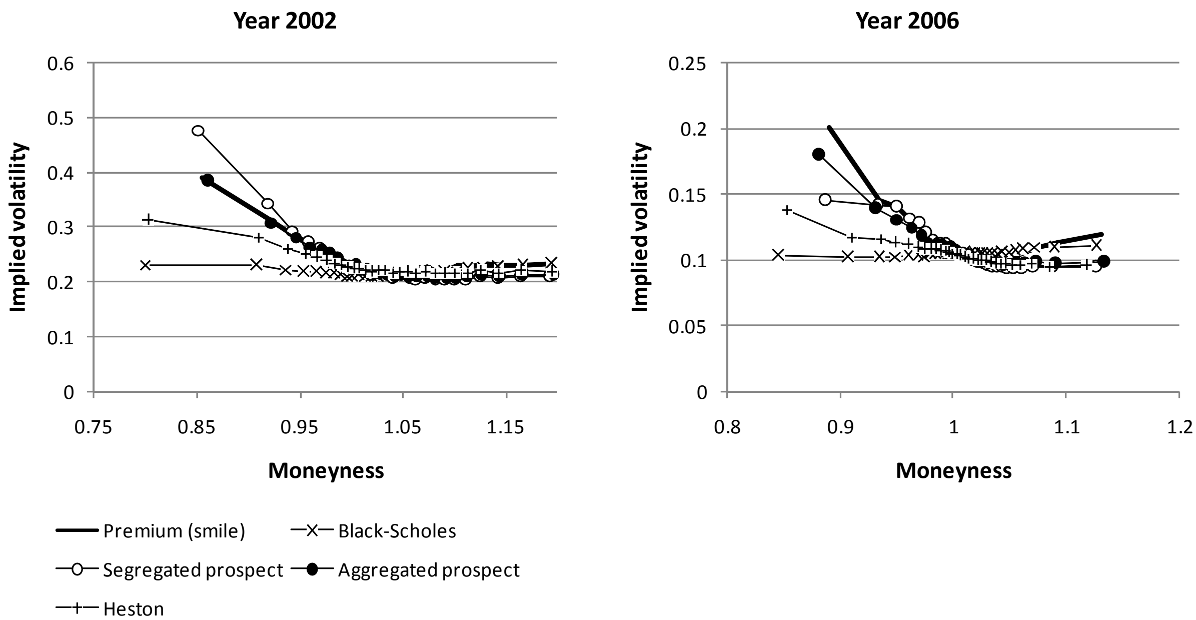


Figure 3: Average implied volatility from S&P 500 call options with relation to *T* (year 2002)

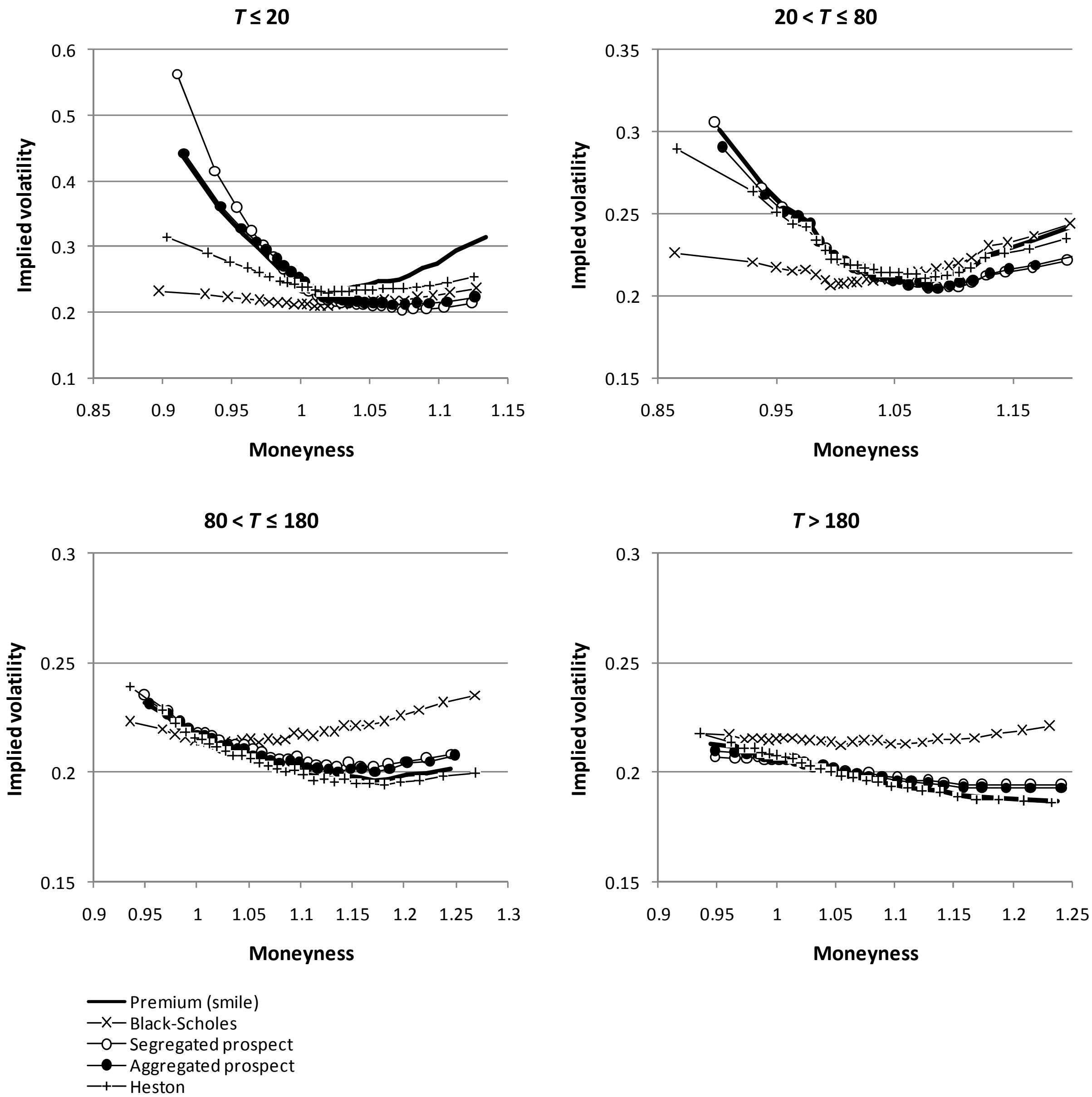


Figure 4: Average implied volatility from S&P 500 call options with relation to *T* (year 2006)

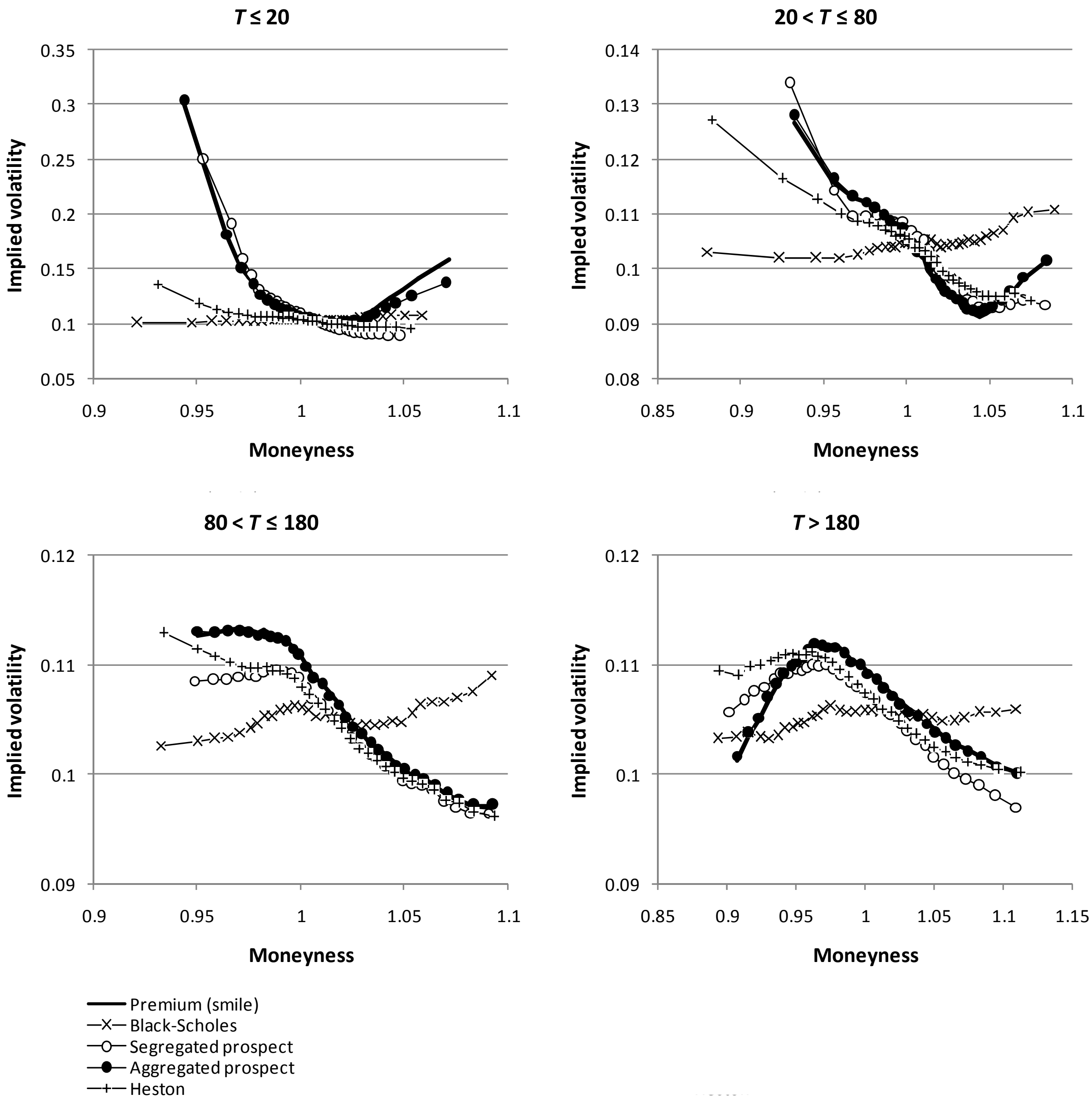


Figure 5: Average mispricing from S&P 500 call options (all data)

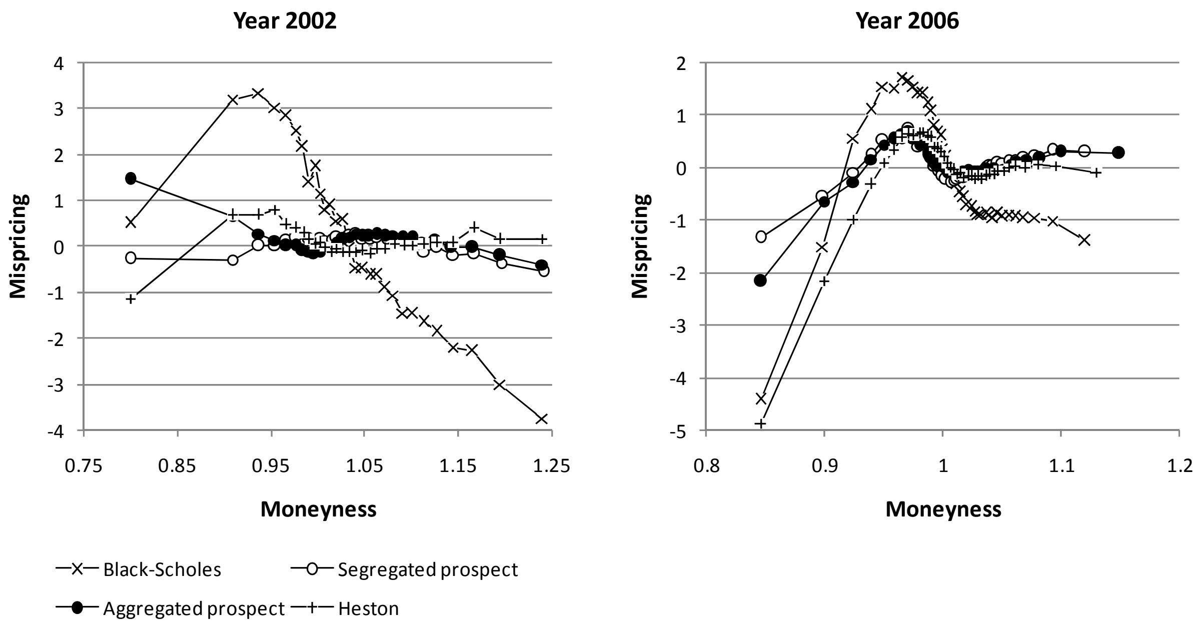


Figure 6: Average mispricing from S&P 500 call options with relation to *T* (year 2002)

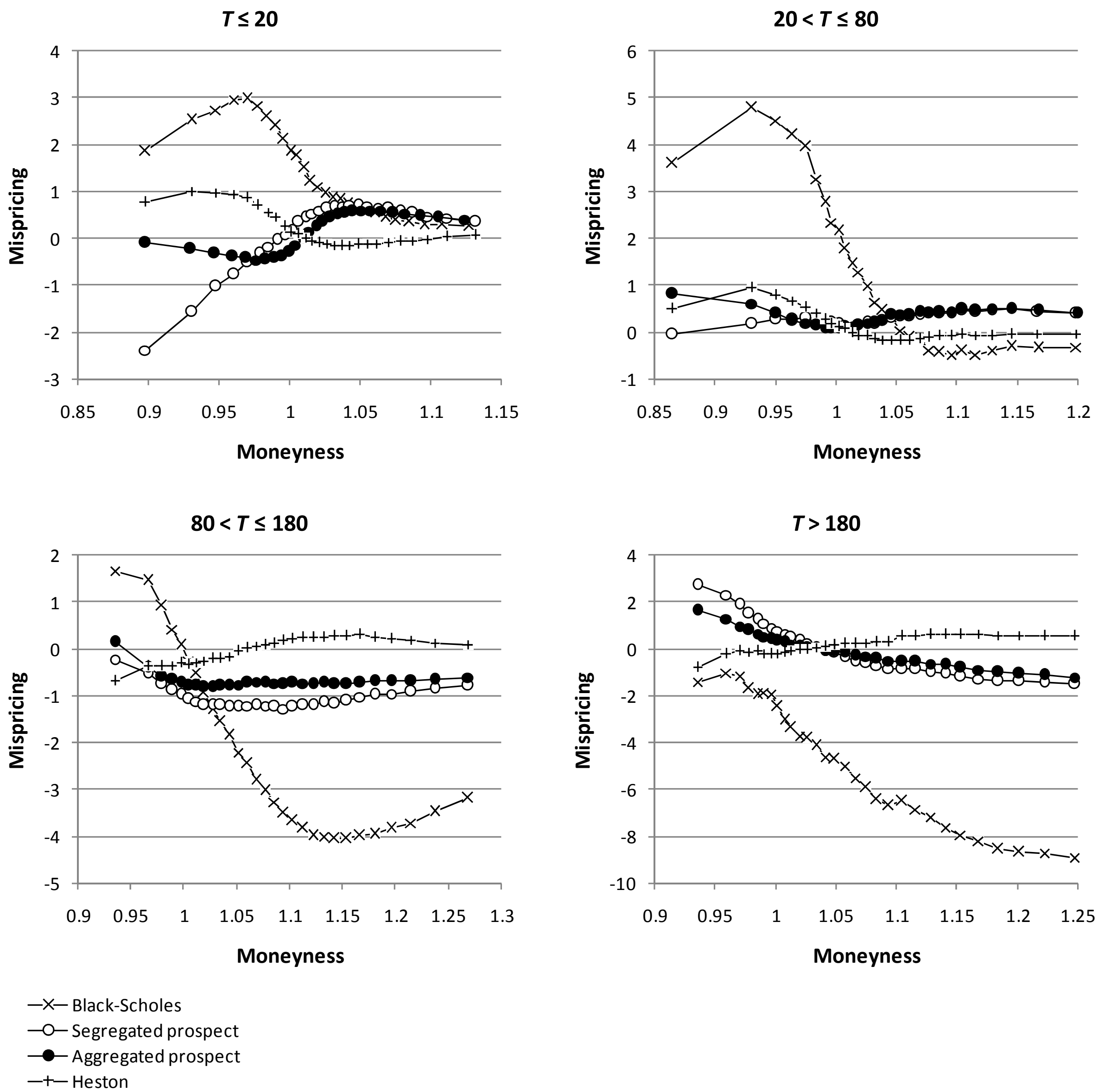


Figure 7: Average mispricing from S&P 500 call options with relation to *T* (year 2006)

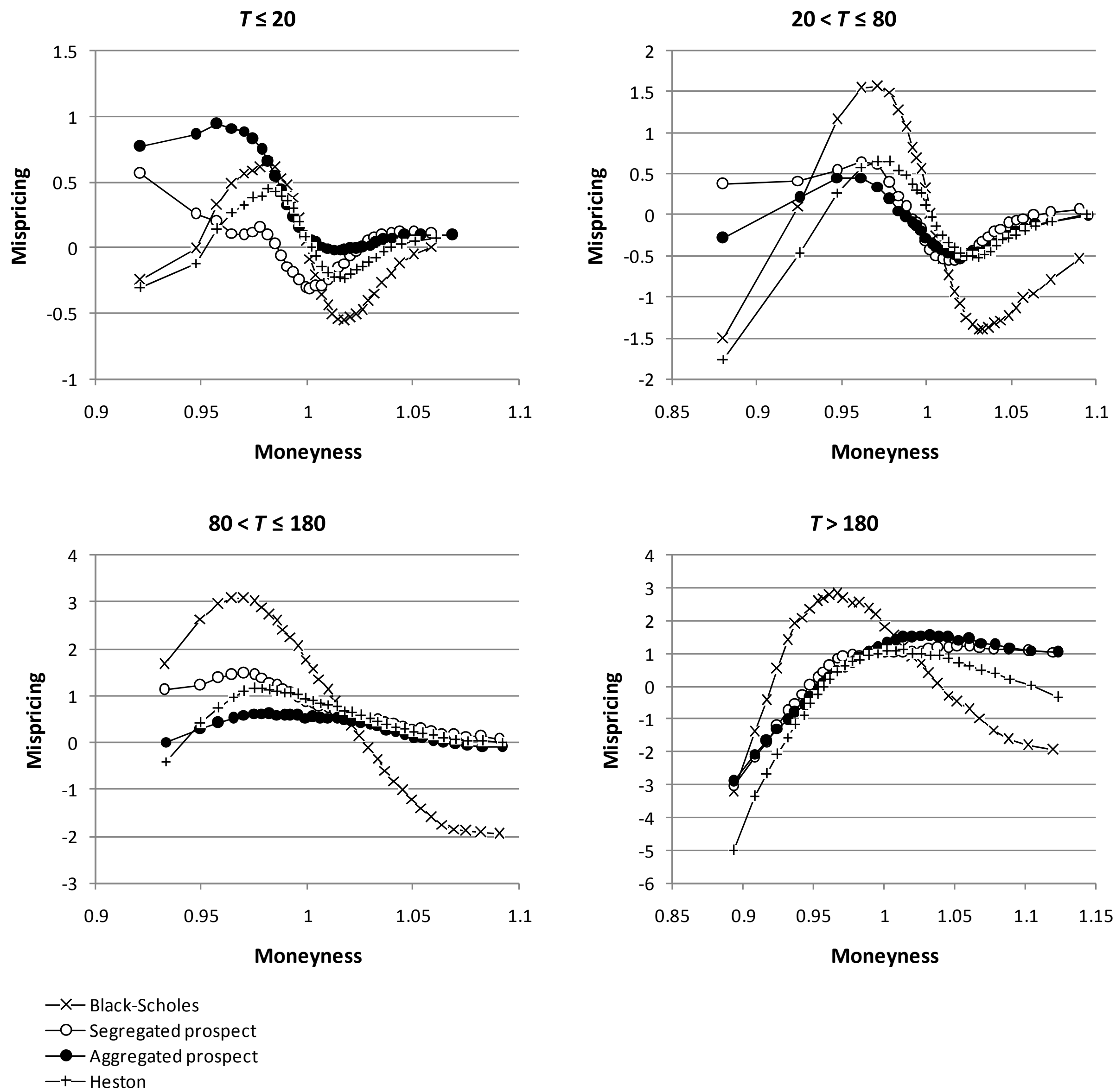


Table 1: Call option prices *

		Tversky-Kahneman sentiment		Moderate sentiment	
K	Black-Scholes	Segregated prospect	Aggregated prospect	Segregated prospect	Aggregated prospect
70	36.72	87.44	48.64	41.43	37.73
80	27.99	66.75	40.28	31.62	29.00
90	19.99	49.18	32.58	22.67	21.00
100	13.27	35.12	25.50	15.17	14.24
110	8.18	24.29	19.38	9.48	9.02
120	4.71	16.21	14.04	5.55	5.34

* Using the following parameters: S_0=100, α=.1, r_f=.1, σ=.2 and T=1 for all models. The Black-Scholes reference has additionally the parameter values a=1, b=1, λ=1 and γ=1. The Tversky-Kahneman sentiment is based on the parameter values, a=.88, b=.88, λ=2.25, γ^+=.61 and γ^-=.69, as reported by Tversky and Kahneman (1992). The moderate sentiment uses the formulas from the prospect theory but relaxes its parameter values to a=.988, b=.988, λ=1.125, γ^+=.961 and γ^-=.969. The value of L is not relevant in these examples.

Table 2: Options data: number of observations

Year 2002

	Time to expiration (trading days)				
Moneyness	T≤20	20<T≤80	80<T≤180	T>180	All
M≤0.9	196	239	96	94	625
0.9<M≤0.925	127	147	35	31	340
0.925<M≤0.95	216	248	66	69	599
0.95<M≤0.975	360	441	132	133	1066
0.975<M≤1	601	759	229	244	1833
1<M≤1.025	716	874	241	177	2008
1.025<M≤1.05	636	736	196	147	1715
1.05<M≤1.075	473	648	214	117	1452
1.075<M≤1.1	374	574	187	121	1256
1.1<M≤1.125	226	469	188	97	980
1.125<M≤1.15	142	291	160	91	684
1.15<M≤1.175	78	224	139	66	507
1.175<M≤1.2	52	164	118	73	407
M>1.2	60	337	410	329	1136
All	4257	6151	2411	1789	14608

Year 2006

	Time to expiration (trading days)				
Moneyness	T≤20	20<T≤80	80<T≤180	T>180	All
M≤0.9	96	355	80	248	779
0.9<M≤0.925	95	162	68	168	493
0.925<M≤0.95	231	389	115	277	1012
0.95<M≤0.975	656	676	267	334	1933
0.975<M≤1	1332	2021	423	230	4006
1<M≤1.025	1471	2185	366	247	4269
1.025<M≤1.05	1039	2089	344	247	3719
1.05<M≤1.075	336	1105	292	183	1916
1.075<M≤1.1	124	379	184	151	838
1.1<M≤1.125	52	105	118	95	370
1.125<M≤1.15	13	37	38	62	150
1.15<M≤1.175	8	36	16	42	102
1.175<M≤1.2	2	22	9	35	68
M>1.2	0	7	17	60	84
All	5455	9568	2337	2379	19739

Table 3: Options data: average call option prices

Year 2002

	Time to expiration (trading days)				
Moneyness	$T \leq 20$	$20 < T \leq 80$	$80 < T \leq 180$	$T > 180$	All
$M \leq 0.9$	140.04	195.91	194.86	213.53	180.88
$0.9 < M \leq 0.925$	85.03	93.66	109.57	145.30	96.79
$0.925 < M \leq 0.95$	64.46	74.91	93.24	126.00	79.04
$0.95 < M \leq 0.975$	42.38	57.26	78.63	108.70	61.30
$0.975 < M \leq 1$	24.10	41.21	64.02	92.77	45.31
$1 < M \leq 1.025$	11.30	28.61	51.16	78.86	29.57
$1.025 < M \leq 1.05$	5.18	18.11	39.58	67.23	19.98
$1.05 < M \leq 1.075$	2.47	11.49	30.32	52.71	14.65
$1.075 < M \leq 1.1$	1.27	7.46	22.34	45.09	11.46
$1.1 < M \leq 1.125$	0.71	5.02	17.14	40.16	9.83
$1.125 < M \leq 1.15$	0.47	3.65	12.77	34.02	9.17
$1.15 < M \leq 1.175$	0.37	2.76	9.91	27.48	7.57
$1.175 < M \leq 1.2$	0.29	2.10	7.06	20.92	6.68
$M > 1.2$	0.25	1.05	3.30	12.00	4.99
All	22.37	30.97	38.70	67.14	34.17

Year 2006

	Time to expiration (trading days)				
Moneyness	$T \leq 20$	$20 < T \leq 80$	$80 < T \leq 180$	$T > 180$	All
$M \leq 0.9$	194.54	174.48	210.90	185.84	184.31
$0.9 < M \leq 0.925$	113.20	114.68	117.67	135.41	121.87
$0.925 < M \leq 0.95$	79.19	83.00	94.53	109.77	90.77
$0.95 < M \leq 0.975$	47.96	53.73	68.36	86.97	59.54
$0.975 < M \leq 1$	20.56	29.81	48.85	67.84	30.93
$1 < M \leq 1.025$	4.32	13.45	31.24	51.76	14.05
$1.025 < M \leq 1.05$	0.60	4.01	18.02	35.98	6.48
$1.05 < M \leq 1.075$	0.13	1.43	9.96	24.82	4.73
$1.075 < M \leq 1.1$	0.07	0.62	4.85	20.40	5.03
$1.1 < M \leq 1.125$	0.06	0.29	2.40	15.87	4.93
$1.125 < M \leq 1.15$	0.03	0.20	1.20	10.09	4.53
$1.15 < M \leq 1.175$	0.08	0.12	0.66	7.28	3.15
$1.175 < M \leq 1.2$	0.05	0.13	0.37	5.24	2.79
$M > 1.2$	-	0.14	0.19	6.05	4.37
All	20.83	26.02	41.27	74.05	32.18

Table 4: Parameters values and wRMSE

Panel A: Black-Scholes model and prospect models

	σ	α	a	b	λ	γ^{+}	γ^{-}	wRMSE
Year 2002								
Black-Scholes	0.215 (0.041)							3.02 (1.61)
Segregated prospect	0.101 (0.020)	-0.019 (0.054)	2.06 (0.68)	2.02 (0.70)	5.73 (45.1)		0.72 (0.20)	0.85 (0.55)
Aggregated prospect	0.359 (0.138)	0.057 (0.082)	0.44 (0.23)	0.41 (0.31)	1.55 (0.70)	0.81 (0.41)	0.94 (0.38)	0.69 (0.43)
Year 2006								
Black-Scholes	0.105 (0.012)							1.36 (0.48)
Segregated prospect	0.045 (0.013)	0.049 (0.009)	2.07 (0.59)	2.05 (0.60)	1.63 (1.89)		0.66 (0.19)	0.62 (0.29)
Aggregated prospect	0.156 (0.057)	0.046 (0.019)	0.66 (0.26)	0.79 (0.30)	0.81 (0.92)	0.95 (0.28)	1.24 (0.34)	0.50 (0.20)

Panel B: Heston model

	κ	θ	ω	ρ	λ	v_0	wRMSE
Year 2002	1.26 (0.77)	0.124 (0.051)	0.51 (0.30)	-0.72 (0.17)	2.1 (1.4)	0.068 (0.042)	0.75 (0.35)
Year 2006	1.35 (0.21)	0.043 (0.006)	0.13 (0.06)	-0.95 (0.13)	3.4 (1.4)	0.011 (0.004)	0.74 (0.37)

Standard deviation within brackets

Table 5: Wilcoxon signed-rank test values of daily (in-sample) wRMSE

Panel A: Year 2002

	Black-Scholes	Segregated prospect	Aggregated prospect	Heston
Black-Scholes	0	13.76 *	13.76 *	13.76 *
Segregated prospect	-13.76 *	0	10.61 *	1.25
Aggregated prospect	-13.76 *	-10.61 *	0	-4.25 *
Heston	-13.76 *	-1.25	4.25 *	0

Panel B: Year 2006

	Black-Scholes	Segregated prospect	Aggregated prospect	Heston
Black-Scholes	0	13.71 *	13.71 *	13.76 *
Segregated prospect	-13.71 *	0	8.96 *	-5.14 *
Aggregated prospect	-13.71 *	-8.96 *	0	-11.08 *
Heston	-13.76 *	5.14 *	11.08 *	0

A negative Wilcoxon value means that the model in the column performs better than the model in the row. And a positive value means the reverse.

* Significant at 1% level

Table 6: In-sample versus out-of-sample wRMSE

	Year 2002		*Year 2006*	
Models	In-sample wRMSE	Out-of-sample wRMSE	In-sample wRMSE	Out-of-sample wRMSE
Black-Scholes	3.03 (1.61)	3.57 (1.91)	1.36 (0.48)	1.66 (0.66)
Segregated prospect	0.85 (0.55)	2.39 (1.88)	0.62 (0.29)	1.49 (0.94)
Aggregated prospect	0.69 (0.43)	1.90 (1.12)	0.50 (0.20)	1.32 (0.77)
Heston	0.75 (0.35)	2.08 (3.18)	0.74 (0.37)	1.33 (0.72)

Standard deviation within brackets

Table 7: Wilcoxon signed-rank test values of daily (out-of-sample) wRMSE

Panel A: Year 2002

	Black-Scholes	Segregated prospect	Aggregated prospect	Heston
Black-Scholes	0	10.11 *	12.46 *	12.44 *
Segregated prospect	-10.11 *	0	8.49 *	8.90 *
Aggregated prospect	-12.46 *	-8.49 *	0	4.17 *
Heston	-12.44 *	-8.90 *	-4.17 *	0

Panel B: Year 2006

	Black-Scholes	Segregated prospect	Aggregated prospect	Heston
Black-Scholes	0	5.69 *	9.07 *	9.98 *
Segregated prospect	-5.69 *	0	5.78 *	4.96 *
Aggregated prospect	-9.07 *	-5.78 *	0	-0.03
Heston	-9.98 *	-4.96 *	0.03	0

A negative Wilcoxon value means that the model in the column performs better than the model in the row. And a positive value means the reverse.

* Significant at 1% level